Accessibilities for Lower-hybrid Wave Propagation*

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Abstract

The damping mechanism of lower-hybrid waves depends on plasma parameters such as plasma density, temperature, magnetic fields, and parallel index of refraction. Wave can propagate to the lower-hybrid resonance layer when the parallel wave-vector is larger than the accessibility criterion vector. Lower-hybrid wave accessibility criterion is evaluated for current drive and heating for ions and electrons

Introduction

- We consider lower-hybrid wave propagation for the damping mechanism which depends on the plasma parameters such as plasma density, temperature, magnetic fields, and parallel index of refraction. Wave can propagate to the lower-hybrid resonance layer when the parallel wave-vector is larger than the accessibility criterion vector ($N_{\parallel \, acc}$). $N_{\parallel \, acc}$ depends on the lower-hybrid wave frequency for the given plasma density and the toroidal magnetic fields.
- Lower-hybrid wave accessibility criterion can be evaluated for current drive and heating for ions and electrons. The lower hybrid wave represents the slow wave branch of the cold plasma dispersion relation in the frequency range $\omega_{ci} << \omega << \omega_{ce}$, where ω is the frequency of the launched wave and ω_{ce} and ω_{ci} are the electron and ion cyclotron frequencies respectively. This branch exhibits a resonance at the lower hybrid frequency. The lower hybrid accessibility criterion is evaluated in this paper.

The Lower-hybrid Accessibility Condition

Wave propagation in a cold plasma is described by the following wave equation for the RF electric field **E**

$$\nabla \times \nabla \times E - \frac{\omega^2}{c^2} K \cdot E = 0$$

where K is the dielectric tensor

$$K = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}$$

where

$$S = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2}$$

$$P = 1 - \frac{\omega_{pe}^2}{\omega^2}$$

$$D = \frac{\omega_{pe}^2}{\omega\omega_{ce}}$$

for $\omega_{ci} << \omega << \omega_{ce}$

The wave equation and full cold plasma dispersion relation

$$N(N \cdot E) - N^2 E + K \cdot E = 0$$

Where $\boldsymbol{N}=c\boldsymbol{k}/\omega$ is the refractive index

Nonzero solution of electric field

$$AN_\perp^4 + BN_\perp^2 + C = 0$$

where

$$\vec{N} = N_\perp \hat{x} + N_\parallel \hat{y}$$

$$N_\perp^2 = N_x^2 + N_y^2$$

$$A = S$$

$$B = (N_\parallel^2 - S)(S + P) + D^2$$

$$C = P[(N_\parallel^2 - S)^2 - D^2]$$

The solution of their dispersion relation relation is given as

$$N_\perp^2 = \frac{B \pm \sqrt{B^2 - 4AC}}{2A}$$

The plus sign tells the slow wave ($N_{\perp s}^2$), and the minus sign tells the fast wave ($N_{\perp f}^2$).

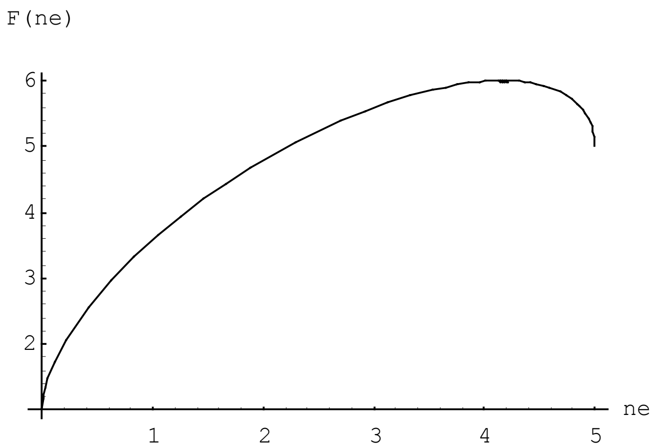
The hybrid resonance occurs when $A \to 0$, and then $N_\perp \to \infty$

$$N_{\perp s}^2 = -\frac{P(N_\parallel^2 - S)}{S} - \frac{D^2}{S} \qquad \omega_{LH} = \omega_{pi}(1 + \frac{\omega_{pe}^2}{\omega_{ce}^2})^{-\frac{1}{2}} \quad [1]$$

$$N_{\perp f}^2 = -\frac{D_\parallel^2 - (N_\parallel^2 - S)^2}{(N_\parallel^2 - S) + \frac{D^2}{P}}$$

S drops linearly from 1 to 0 with increasing density ($0 \le S \le 1$)

S is linear zero at the resonance, we can write $S = 1 - n_e / n_e^{(res)}$, where n_e is the plasma density.



$$S[(n_e)_{\max}] = 1 - \frac{(n_e)_{\max}}{n_e^{(res)}} = \frac{1}{1 + n_e^{(res)}}$$

(1) The sufficiently low densities : $|D| \langle\langle \; 1 \quad S \cong 1$

$$(N_\perp^2 - P(1 - N_\parallel))(N_\perp^2 - (1 - N_\parallel^2)) = 0$$

$$(N_\perp^2 - P(1 - N_\parallel)) = 0$$

The accessibility condition is always $N_{||}^2 > 1$

P = 1 (zero density) evanescent wave.

P < 0 (negative) propagating wave

P = 0 layer occurs at an extremely low value of electron density. The evanescent region in the presence of plasma is therefore very thin and the wave attenuation is very slight at the low hybrid frequency.

At the region of plasma away from the edge.

$$N_{\parallel}^{2} > \left|\frac{RL}{P}\right| + |S|$$

$$where \quad B = RL + PS - PN_{\parallel}^{2} - SN_{\parallel}^{2} > 0 \qquad |P| \rangle\rangle S \cong 1 \qquad RL = S^{2} - D^{2} > 0$$

This condition is satisfied automatically if we satisfy the middle condition $B^2 - 4AC > 0$

$$N_{\parallel}^{2} > (S^{\frac{1}{2}} + \left|\frac{D^{2}}{P}\right|^{\frac{1}{2}})^{2}$$

Therefore we have only to consider the condition $B^2 - 4AC > 0$ for the wave propagation. ($N_{\perp}^{2} > 0$)

For the lower hybrid range of frequencies $\omega_{ci} << \omega << \omega_{ce}$ and $\omega \sim \omega_{pi}$ $(or\ \omega > \omega_{LH})$

$$N_{\parallel acc} = (1 + \frac{\omega_{pe}^{2}}{\omega_{ce}^{2}} - \frac{\omega_{pi}^{2}}{\omega})^{\frac{1}{2}} + \frac{\omega_{pe}}{\omega_{ce}}$$ [3] at the low density

from $N_{\parallel}^{2} = (S^{\frac{1}{2}} + \left|\frac{D^{2}}{P}\right|^{\frac{1}{2}})^{2} = N_{\parallel acc} \qquad (B^{2} - 4AC = 0)$

(2) The high density region

The right hand side of $N_{ll}^2 > (S^{\frac{1}{2}} + \left|\frac{D^2}{P}\right|^{\frac{1}{2}})^2$ has the maximum value

$$F(n_e) = \left[(1-\frac{n_e}{n_e^{\mathrm{res}}})^{\frac{1}{2}} + n_e^{1/2}\right]^2 \quad \text{where} \quad D \sim P \sim n_e \quad and \quad S = 1 - \frac{n_e}{n_e^{res}} \quad (n_e : \text{electron density})$$

S is zero at the resonance

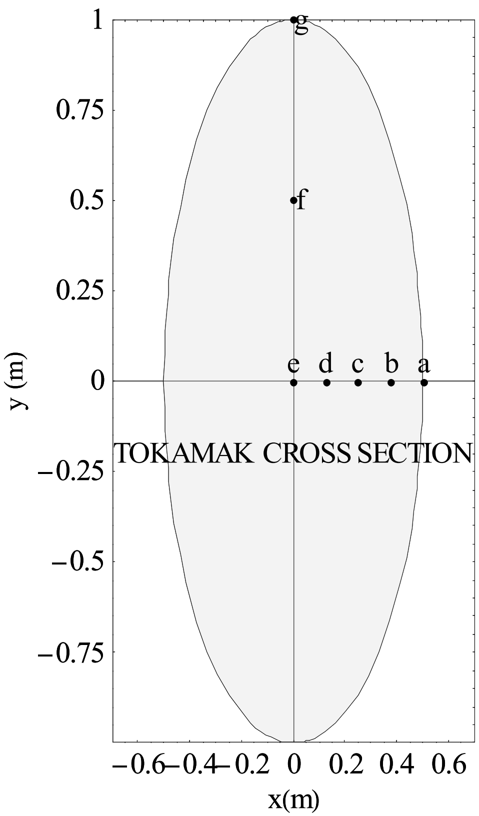
$$F(n_e) \cong 1 + \left|\frac{D^2}{P}\right|_{(\mathrm{res})} \quad Thus \quad N_{\|\mathrm{acc}}^2 > 1 + \left|\frac{D^2}{P}\right|_{(\mathrm{res})} = \left[1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\right]_{\omega=\omega_{\mathrm{LH}}}$$

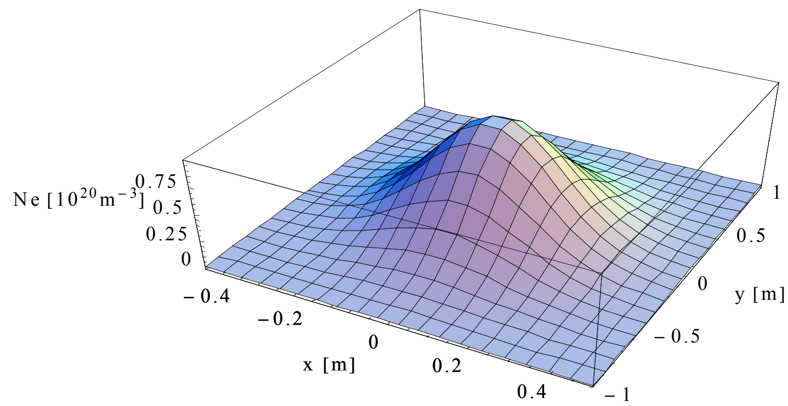
at the lower hybrid region $\omega_{\mathrm{ci}} << \omega << \omega_{ce}$

Lower-hybrid wave accessibility criterion [3]

$$N_{\|\mathrm{acc}}^2 > \left[1 + \frac{\omega_{pe}^2}{\omega_{ce}^2}\right]_{\omega = \omega_{\mathrm{LH}}}$$

Plasma density and toroidal B field profiles in tokamak





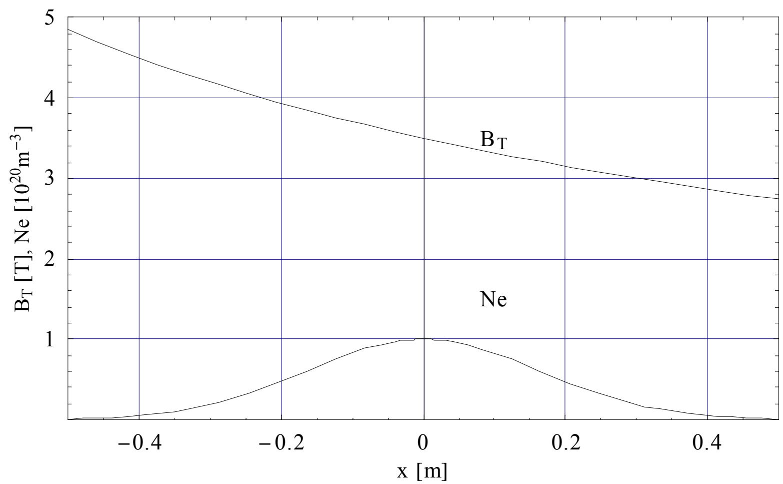


Figure 1.

$$n_e(r) = n_{e0} Exp(-\alpha r^2) \quad B_t(x) = B_0 \frac{R_0}{R_0 + x} \quad (n_{e0} = 1.0 \times 10^{20} m^{-3},\ \alpha = 18.42,\ R_0 = 1.8m,\ B_0 = 3.5T,\ r^2 = x^2 + 0.5y^2)$$

Lower Hybrid wave penetration and accessibility

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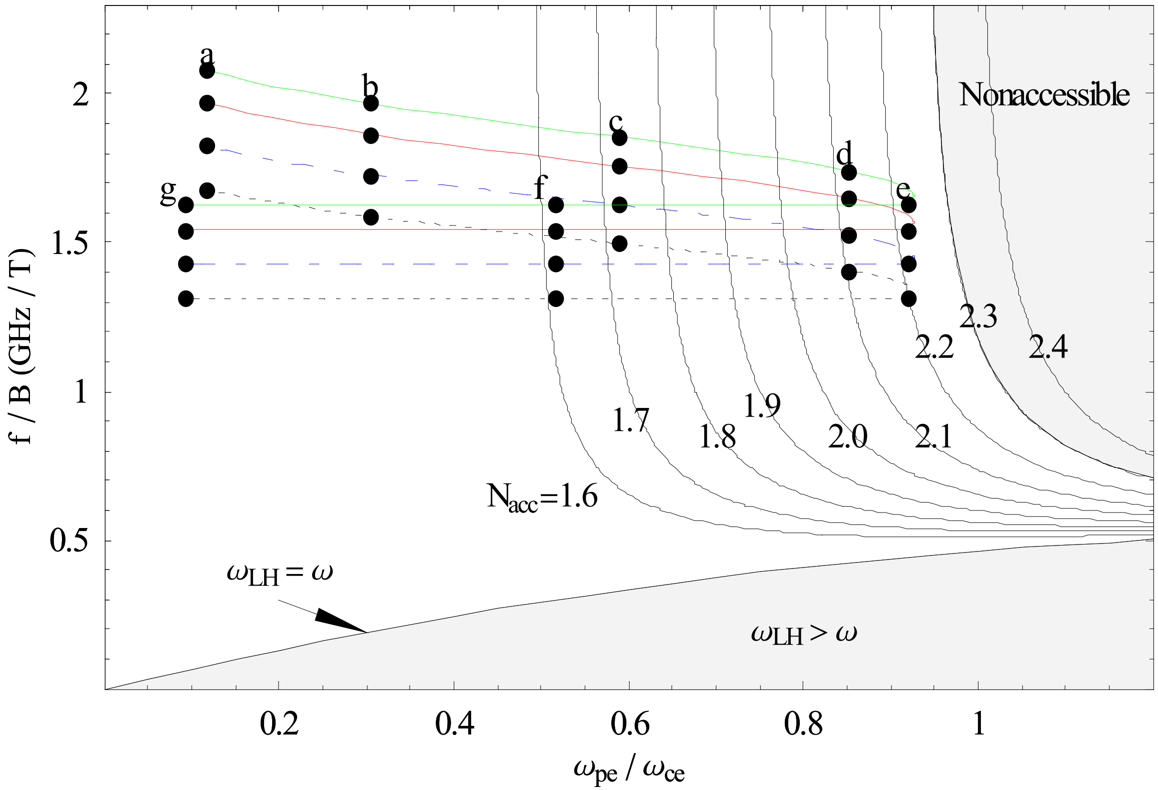

$$\frac{\omega_{LH}}{\omega} = \left[\frac{\omega_{pi}^2/\omega^2}{1+\omega_{pi}^2/\omega_{ce}^2}\right]^{\frac{1}{2}}$$

$$N_{llacc} = \frac{\omega_{pe}}{\omega_{ce}} + \left[1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2}\right]^{\frac{1}{2}}$$

Let $u = \omega_{pe}/\omega_{ce}$ and $\upsilon = f/B_t$

$$\frac{\omega_{LH}}{\omega} = \left[\frac{784\,u^2}{\gamma^2 A\,\upsilon^2(1+u^2)}\right]^{\frac{1}{2}}$$

$$N_{llacc} = u + \left[1 + u^2 - \frac{784\,u^2}{\gamma A\,\upsilon^2}\right]^{\frac{1}{2}}$$

Figure 2. The $f\,/\,B$ vs. ω_{pe}/ω_{ce}, where f is the LH launched frequency, ω_{pe} is the plasma frequency, and ω_{ce} is the electron cyclotron frequency. This figure shows the accessible $N_{||acc}$ values from 1.6 to 2.4, the positions (a-b-c-d-e-f-g), and $\omega_{LH}/\,\omega = 1$ line as a function of $f\,/\,B$ and $\omega_{pe}/\,\omega_{ce.}$ The lower hybrid frequency ω_{LH} and $N_{||acc}$ depend on plasma density and toroidal B fields. The green solid line corresponds to 5.7 GHz, and the red solid line to 5.4 GHz, the blue dash line to 5.0 GHz, the black dash line to 4.6 GHz. The “a” is the plasma limiter position (x=50 cm) and “e” is the tokamak center (x=0.0) in horizontal direction, and the “f and g” are the vertical positions as shown in the previous graph. When the launched frequency ω is smaller than the lower hybrid frequency, ion heating occurs. But, the launched frequencies of 4.6 GHz, 5.0 GHz, 5.4 GHz, 5.7 GHz are all lager than the lower hybrid frequency ω_{LH} with a given tokamak plasma and toroidal B field profiles as shown in the previous graphs.

Conclusions

• The lower hybrid wave can be injected with $N_{||}$ spectrum in some range by adjusting the phase shift between wave-guides in the antenna. But the $N_{||}$ must be lager than $N_{||acc}$ (the lower hybrid wave accessibility criterion) for the lower hybrid wave penetration through the tokamak plasma. $N_{||acc}$ depends on the launched wave- frequency for the given plasma density and the toroidal magnetic field. $N_{||acc}$ becomes large for the launched wave-frequency.

• For the values of $N_{||} \geq N_{||acc}$, the slow wave which is a root of the dispersion equation can propagate to the lower hybrid resonance layer. If the lower hybrid resonance ($\omega_{ci} << \omega << \omega_{ce}$) is present in the plasma, the accessibility criterion reduces to

$$N_{llacc}^{2} > \left[1 + \frac{\omega_{pe}^{2}}{\omega_{ce}^{2}} \right]_{\omega = \omega_{LH}}$$

When the lower hybrid resonance does not exist in the plasma ($\omega > \omega_{LH}$), electron heating and the current drive may still take place.

References

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[3] Stix, T. H., The Theory of Plasma Waves, pp 4 ~ 104 (1992).