7 Calculation of ECR optical depth using Mathematica

Calculation of Optical Depth for EC-wave Heating

```
<< Graphics`Arrow`;
<< Graphics`Colors`;
<< Graphics`FilledPlot`;
<< Graphics`Graphics`

Off[General::spell];
Off[General::spell1];

$TextStyle = {FontFamily → "Times", FontSize → 16};

R0 = 180;
a = 50;
f = 84;
w = 2 Pi f;
fpe = 90 Sqrt[Ne0 (1 - rho^2)];
fce[n_] = f / (n (1 + (a / R0) rho));
alpha = (fpe / f)^2;
alphan[n_] = (fce[n] / f)^2;
lambda = 0.3;
mc2 = 511;
Te0 = 10;
```

For Fundamental Harmonic

```
Nx1stX = 0.5 (3 - Nz^2 - 2 alpha + Sqrt[(1 + Nz^2)^2 - 4 alpha Nz^2]);
Nx1stO = 0.5 (3 - Nz^2 - 2 alpha - Sqrt[(1 + Nz^2)^2 - 4 alpha Nz^2]);
```

(+) sign: X-mode like, (--) sign: O-mode like (*Opposite sense to the signs of the solutions of the cold plasma dispersion relation*)

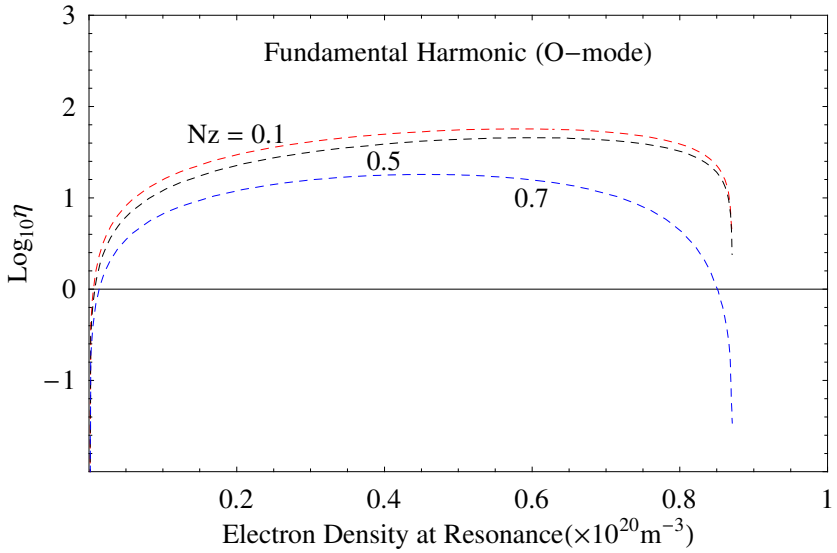
```
etaFundO = 16 Pi / alpha R0 / lambda Nz^2 / Sqrt[Nx1stO]
    Te0 / mc2 ((1 - alpha / 2 - Nz^2) (1 - alpha) - Nx1stO)^2 /
     ((1 - alpha - Nx1stO)^2 + (1 - alpha) Nz^2);
etaFundX = 16 Pi / alpha R0 / lambda Nz^2 / Sqrt[Nx1stX] Te0 / mc2
    ((1 - alpha / 2 - Nz^2) (1 - alpha) - Nx1stX)^2 /
     ((1 - alpha - Nx1stX)^2 + (1 - alpha) Nz^2);

FundOmode1 = Plot[Log[10, etaFundO] //. {Nz → 0.1, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}], RGBColor[1, 0, 0]}]
```

```
FundOmode2 = Plot[Log[10, etaFundO] //. {Nz → 0.5, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}]}]

FundOmode3 = Plot[Log[10, etaFundO] //. {Nz → 0.7, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}], RGBColor[0, 0, 1]}]

Show[FundOmode1, FundOmode2, FundOmode3, Graphics[
  {Text["Nz = 0.1", {0.2, 1.7}], Text["0.5", {0.4, 1.4}], Text["0.7", {0.6, 1}],
   Text["Fundamental Harmonic (O-mode)", {0.5, 2.6}]}], PlotRange → {{0, 1}, {-2, 3}},
 FrameLabel → {"Electron Density at Resonance(x10^20m^-3)", "Log10η"}, Frame → True]
```



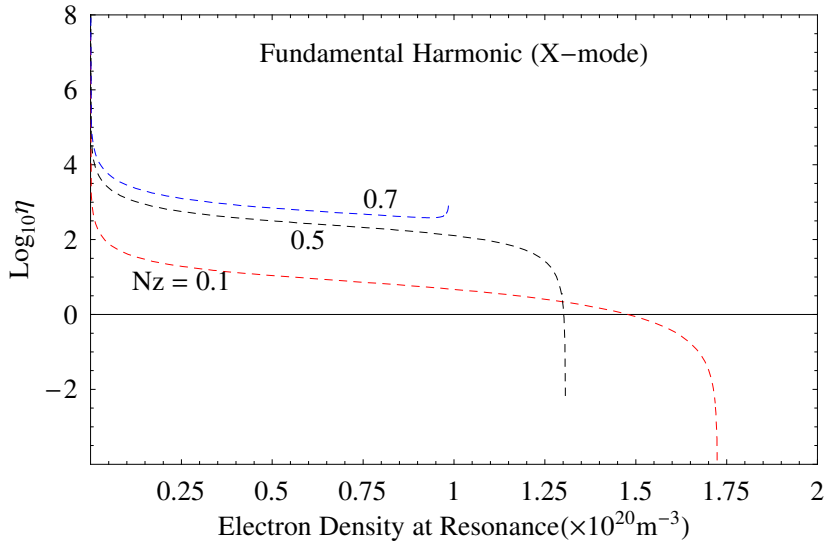

```
- Graphics -

FundXmode1 = Plot[Log[10, etaFundX] //. {Nz → 0.1, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}], RGBColor[1, 0, 0]}]

FundXmode2 = Plot[Log[10, etaFundX] //. {Nz → 0.5, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}]}]

FundXmode3 = Plot[Log[10, etaFundX] //. {Nz → 0.7, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}], RGBColor[0, 0, 1]}]
```

```
Show[FundXmode1, FundXmode2, FundXmode3, Graphics[
  {Text["Nz = 0.1", {0.25, 0.9}], Text["0.5", {0.6, 2}], Text["0.7", {0.8, 3.1}],
   Text["Fundamental Harmonic (X-mode)", {1, 7}]}], PlotRange → {{0, 2}, {-4, 8}},
 FrameLabel → {"Electron Density at Resonance(×10^20m^-3)", "Log10η"}, Frame → True]
```

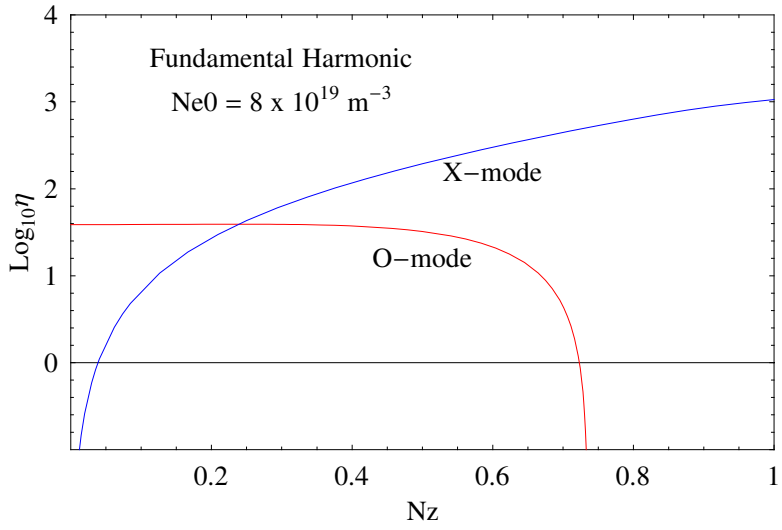


- Graphics -

```
FundOmode4 = Plot[Log[10, etaFundO] //. {Ne0 → 0.8, rho → 0},
  {Nz, 0., 1}, PlotStyle → {RGBColor[1, 0, 0]}]
```

```
FundXmode4 = Plot[Log[10, etaFundX] //. {Ne0 → 0.8, rho → 0},
  {Nz, 0., 1}, PlotStyle → {RGBColor[0, 0, 1]}]
```

```
Show[FundOmode4, FundXmode4,
 Graphics[{Text["O-mode", {0.5, 1.2}], Text["X-mode", {0.6, 2.2}],
   Text["Fundamental Harmonic", {0.3, 3.5}], Text["Ne0 = 8 x 10^19 m^-3", {0.3, 3}]}],
 PlotRange → {{0, 1}, {-1, 4}}, FrameLabel → {"Nz", "Log10η"}, Frame → True]
```



- Graphics -

For Higher Harmonics

■ Second Harmonic

```
SS[n_] = 1 - alpha / (1 - alphan[n]);
PP = 1 - alpha;
DD[n_] = -alpha Sqrt[alphan[n]] / (1 - alphan[n]);

AA = SS[n];
BB = -(SS[n] + PP) (SS[n] - Nz^2) + DD[n]^2;
CC = PP ((SS[n] - Nz^2)^2 - DD[n]^2);

Disc = Sqrt[BB^2 - 4 AA CC];

NxO = (-BB + Disc) / (2 AA);
NxX = (-BB - Disc) / (2 AA);

NO2 = Nz^2 + NxO;
NX2 = Nz^2 + NxX;
```

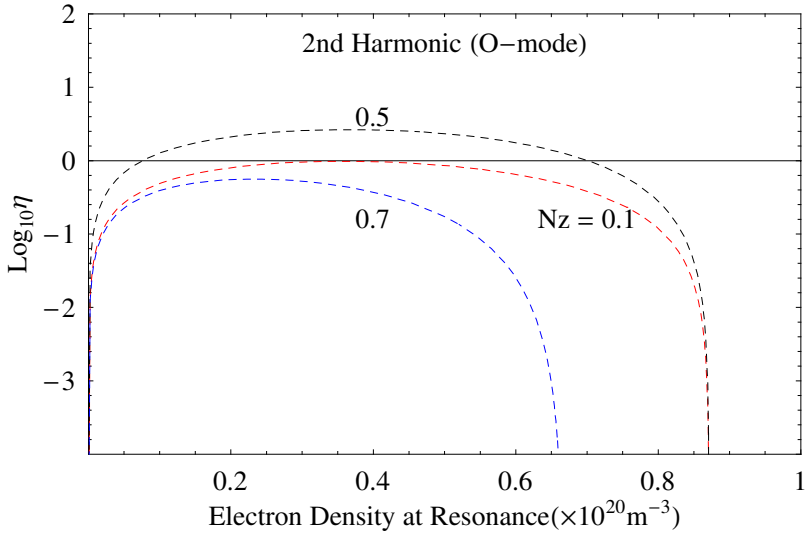
```
etaHarmO[n_] = 2 Pi R0 / lambda Te0 / mc2 (NxO n^2 Te0 / (2 mc2))^(n - 2)
    n^3 / Factorial[n - 1] alpha Sqrt[NxO] ((SS[n] - DD[n] - NO2)^2 (PP - NxO)^2) /
     (DD[n]^2 (PP - NxO)^2 + PP Nz^2 (SS[n] - NO2)^2);
etaHarmX[n_] = 2 Pi R0 / lambda Te0 / mc2 (NxX n^2 Te0 / (2 mc2))^(n - 2)
    n^3 / Factorial[n - 1] alpha Sqrt[NxX] ((SS[n] - DD[n] - NX2)^2 (PP - NxX)^2) /
     (DD[n]^2 (PP - NxX)^2 + PP Nz^2 (SS[n] - NX2)^2);

SecondOmode1 = Plot[Log[10, etaHarmO[2]] //. {Nz → 0.1, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}], RGBColor[1, 0, 0]}]

SecondOmode2 = Plot[Log[10, etaHarmO[2]] //. {Nz → 0.5, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}]}]

SecondOmode3 = Plot[Log[10, etaHarmO[2]] //. {Nz → 0.7, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}], RGBColor[0, 0, 1]}]

Show[SecondOmode1, SecondOmode2, SecondOmode3, Graphics[
  {Text["Nz = 0.1", {0.7, -0.8}], Text["0.5", {0.4, 0.6}], Text["0.7", {0.4, -0.8}],
   Text["2nd Harmonic (O-mode)", {0.5, 1.6}]}], PlotRange → {{0, 1}, {-4, 2}},
 FrameLabel → {"Electron Density at Resonance(×10^20m^-3)", "Log10η"}, Frame → True]
```



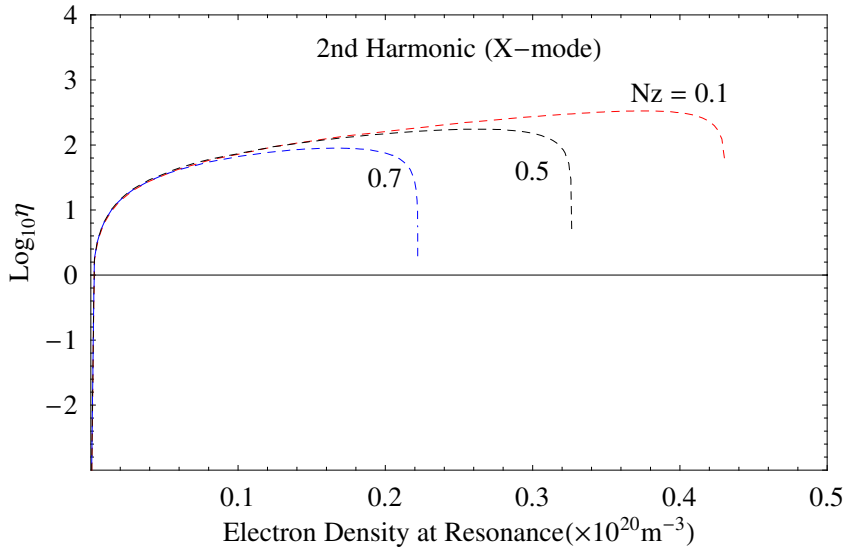

```
- Graphics -
```

```
SecondXmode1 = Plot[Log[10, etaHarmX[2]] //. {Nz → 0.1, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}], RGBColor[1, 0, 0]}]

SecondXmode2 = Plot[Log[10, etaHarmX[2]] //. {Nz → 0.5, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}]}]

SecondXmode3 = Plot[Log[10, etaHarmX[2]] //. {Nz → 0.7, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}], RGBColor[0, 0, 1]}]
```

```
Show[SecondXmode1, SecondXmode2, SecondXmode3, Graphics[
  {Text["Nz = 0.1", {0.4, 2.8}], Text["0.5", {0.3, 1.6}], Text["0.7", {0.2, 1.5}],
   Text["2nd Harmonic (X-mode)", {0.25, 3.5}]}], PlotRange → {{0, 0.5}, {-3, 4}},
 FrameLabel → {"Electron Density at Resonance(×10^20m^-3)", "Log10η"}, Frame → True]
```

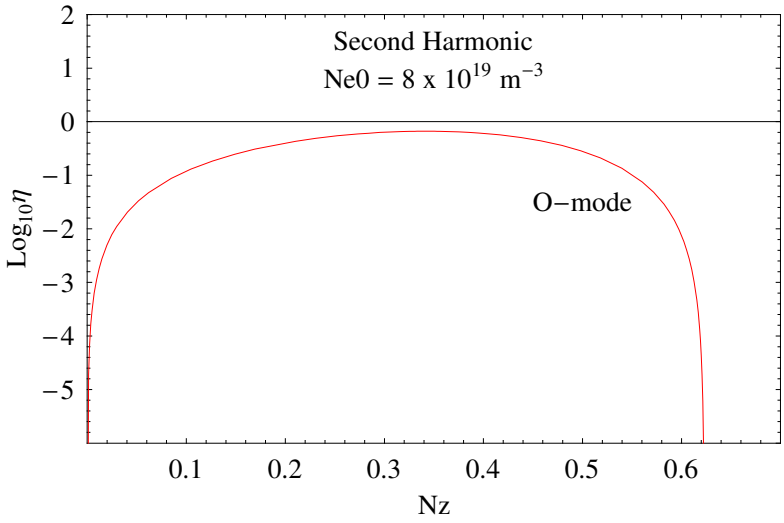



```
- Graphics -
```

```
SecondOmode4 = Plot[Log[10, etaHarmO[2]] //. {Ne0 → 0.8, rho → 0},
  {Nz, 0., 1}, PlotStyle → {RGBColor[1, 0, 0]}]
```

```
SecondXmode4 = Plot[Log[10, etaHarmX[2]] //. {Ne0 → 0.4, rho → 0},
  {Nz, 0.0, 1}, PlotStyle → {RGBColor[0, 0, 1]}]
```

```
Show[SecondOmode4, Graphics[{Text["O-mode", {0.5, -1.5}],
   Text["Second Harmonic", {0.35, 1.5}], Text["Ne0 = 8 x 10^19 m^-3", {0.35, 0.8}]}],
 PlotRange → {{0, 0.7}, {-6, 2}}, FrameLabel → {"Nz", "Log10η"}, Frame → True]
```




```
- Graphics -
```

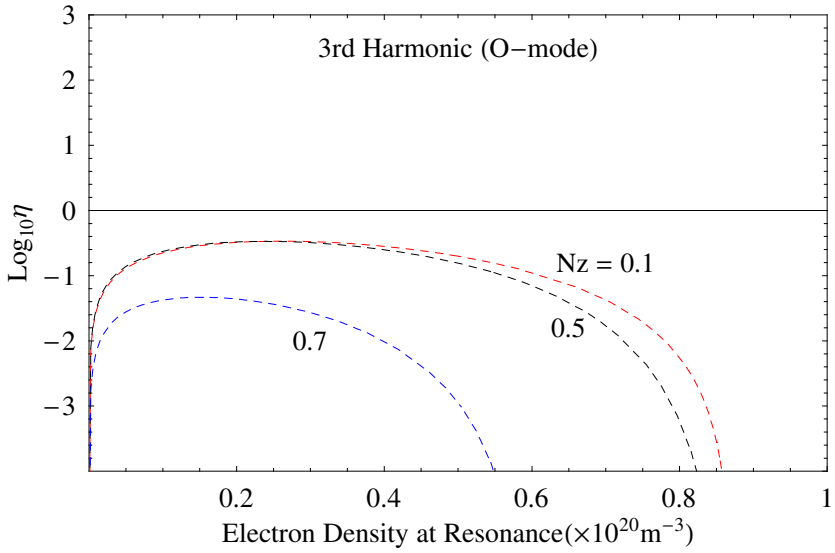
■ Thrid Harmonic

```
ThirdOmode1 = Plot[Log[10, etaHarmO[3]] //. {Nz → 0.1, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}], RGBColor[1, 0, 0]}]

ThirdOmode2 = Plot[Log[10, etaHarmO[3]] //. {Nz → 0.5, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}]}]

ThirdOmode3 = Plot[Log[10, etaHarmO[3]] //. {Nz → 0.7, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}], RGBColor[0, 0, 1]}]
```

```
Show[ThirdOmode1, ThirdOmode2, ThirdOmode3, Graphics[
  {Text["Nz = 0.1", {0.7, -0.8}], Text["0.5", {0.65, -1.8}], Text["0.7", {0.3, -2}],
   Text["3rd Harmonic (O-mode)", {0.5, 2.5}]}], PlotRange → {{0, 1}, {-4, 3}},
 FrameLabel → {"Electron Density at Resonance(×10^20m^-3)", "Log10η"}, Frame → True]
```



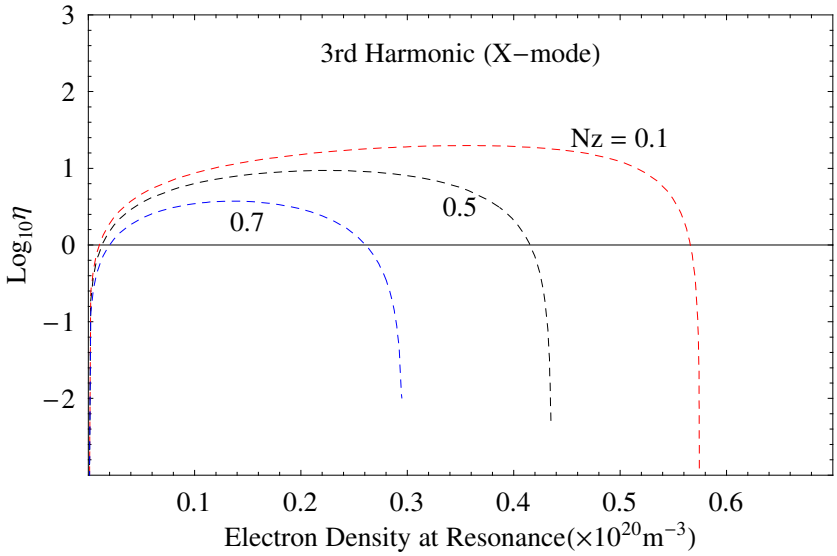
- Graphics -

```
ThirdXmode1 = Plot[Log[10, etaHarmX[3]] //. {Nz → 0.1, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}], RGBColor[1, 0, 0]}]

ThirdXmode2 = Plot[Log[10, etaHarmX[3]] //. {Nz → 0.5, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}]}]

ThirdXmode3 = Plot[Log[10, etaHarmX[3]] //. {Nz → 0.7, rho → 0},
  {Ne0, 0.0, 2}, PlotStyle → {Dashing[{0.01, 0.01}], RGBColor[0, 0, 1]}]
```

```
Show[ThirdXmode1, ThirdXmode2, ThirdXmode3, Graphics[
  {Text["Nz = 0.1", {0.5, 1.4}], Text["0.5", {0.35, 0.5}], Text["0.7", {0.15, 0.3}],
   Text["3rd Harmonic (X-mode)", {0.35, 2.5}]}], PlotRange → {{0, 0.7}, {-3, 3}},
 FrameLabel → {"Electron Density at Resonance(×10^20 m^-3)", "Log10 η"}, Frame → True]
```



- Graphics -