2 EC-wave propagation using Mathematica

2.1 O-X propagation for 2nd harmonic resonance for KSTAR tokamak with low density

Propagation of 84-GHz Microwave in KSTAR tokamak for Second Harmonic Resonance with low plasma density

- **Electron density in unit of 10^20 m^-3**
  Electron temperuture in unit of keV
  Toroidal magnetic field in unit of Tesla
  All frequencies in unit of GHz

  KSTAR major radius: 1.8 m
  KSTAR plasma minor radius: 0.5 m
  KSTAR toroidal magnetic field, B0: 1.5 T
  KSTAR ECH system frequency: 84 GHz

```
Clear["Global`*"]
```

```
Off[General::spell];
Off[General::spell1];
```

```
a = 0.5;
R0 = 1.8;
f = 84.0;
bz0 = 1.5;
te0 = 10.;
ne1 = 0.;
Nh = 2;
Nnu = 1.;
Tnu = 1;
Az = 1;
massr = 2000. Az;
sc = 3 10^8;
mc2 = 511.0;
```

```
ne = ne0 ((1 - rho^2)) ^Nnu + ne1;
te = te0 ((1 - rho^2)) ^Tnu;
ve = sc Sqrt[2 te / mc2];
bz = bz0 / (1 + (a / R0) rho);
fce = 28.0 bz;
fci = fce / massr;
fpe = 90.0 Sqrt[ne];
fpi = fpe / Sqrt[massr];
w = 2.0 Pi f;
wce = 2.0 Pi fce;
wpe = 2.0 Pi fpe;
wci = 2.0 Pi fci;
wpi = 2.0 Pi fpi;
```

- **SS = 1 -** $\omega^2_{pe} / (\omega^2 - \omega^2_{ce})$
  DD = $(-\omega^2_{pe}/(\omega^2 - \omega^2_{ce}))\ (\omega_{ce}/\omega)$
  PP = 1 - ω^2_{pe}/ω^2

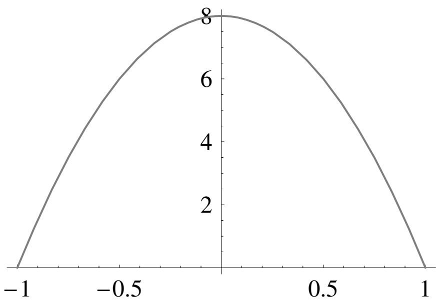
  If define
  q = $\omega^2_{pe}/\ \omega^2$
  u = $\omega^2_{ce}/\ \omega^2$

```
q = wpe^2 / w^2;
u = wce^2 / w^2;

SS = 1 - q / (1 - u);
DD = -q / (1 - u) Sqrt[u];
PP = 1 - q;

AA = SS;
BB = - (SS + PP) (SS - Npar^2) + DD^2;
CC = PP ((SS - Npar^2)^2 - DD^2);

Disc = BB BB - 4 AA CC;

Nppsq = (-BB + Sqrt[Disc]) / (2.0 AA);
Npnsq = (-BB - Sqrt[Disc]) / (2.0 AA);
```

▪ + sign : O-mode , - sign : X-mode

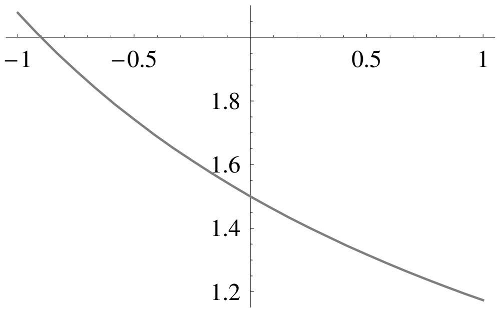
▪ Density plot (low density)

```
NePlot = Plot[10 ne /. ne0 → 0.8, {rho, -1, 1},
  PlotStyle → {Thickness[0.005], GrayLevel[0.5]}]
```




```
- Graphics -
```

```
BTPlot = Plot[ bz, {rho, -1, 1}, PlotStyle → {Thickness[0.005], GrayLevel[0.5]}]
```




```
- Graphics -
```

■ Cutoff and Resonances

O-mode cutoff: q = 1 **(⟸ P = 0)**

X-mode cutoff: $q = (1-\sqrt{u})(1-N_{\parallel}^2)$ **(⟸ $(S-N_{\parallel}^2)^2 - D^2 = 0$)**

Upper Hybrid Resonance: q = 1 - u **(⟸ $\omega^2 = \omega^2_{pe} + \omega^2_{ce}$)**

Electron Cyclotron Resonance: $\sqrt{u} = 1$ **(⟸ $\omega = N\,\omega_{ce}$)**

O-X conversion: $q = 1 + u\,[(1-N_{\parallel}^2)/(2\,N_{\parallel})]^2$ **(⟸ $B^2 - 4\,AC = 0$)**

```
Clear[Npar];
```

```
ocut = 1;
xcut = (1 - Sqrt[u]) (1 - Npar^2);
uhr = (1 - u);
ecr = Nh Sqrt[ u] - 1;
oxc = 1 + u ((1 - Npar^2) / (2 Npar)) ^2;
```

- **O-Mode Cutoff position (for maximum density of 0.8 x 10^20 m^-3)**

```
solocut = Solve[q == ocut //. ne0 → 0.8, rho]
{{rho → 0. - 0.298142 i}, {rho → 0. + 0.298142 i}}
```

→ *No O-mode cutoff!*

- **X-mode Cutoff position for $N_{\parallel} = 0.5$**

```
solxcut = Solve[q == xcut //. {ne0 → 0.8, Npar → 0.5}, rho]
{{rho → -3.70834}, {rho → -3.4765}, {rho → -0.846925}, {rho → 0.723424}}

xcutrho1 = rho /. solxcut[[4]];
xcutrho2 = rho /. solxcut[[3]];
```

- **Upper Hybrid Resonance position**

```
soluhr = Solve[q == uhr /. ne0 → 0.8, rho]
{{rho → -4.06125}, {rho → -2.97094}, {rho → -0.535528}, {rho → 0.367715}}

uhrrho1 = rho /. soluhr[[4]];
uhrrho2 = rho /. soluhr[[3]];
```

- **ECR position**

```
solecr = Solve[ecr == 0, rho]
{{rho → -7.2}, {rho → 0.}}

ecrrho = rho /. solecr[[2]];
```

- **O-X Conversion [No O-X conversion]**

- **[1] For fixed centeral density, n_{e0}=0.8x $10^{20}\, m^{-3}$ and fixed $N_{\parallel} = 0.5$**

```
soloxc1 = Solve[q == oxc //. {ne0 → 0.8, Npar → 0.5}, rho]
{{rho → -3.63975 - 0.38162 i}, {rho → -3.63975 + 0.38162 i},
 {rho → 0.0397483 - 0.482289 i}, {rho → 0.0397483 + 0.482289 i}}
```

→ *No solution!*

- **[2] For fixed centeral density, n_{e0}=0.8 x $10^{20}\, m^{-3}$ and fixed $N_{||}$ = 0.7**

```
soloxc2 = Solve[q == oxc //. {ne0 → 0.8, Npar → 0.7}, rho]
```

```
{{rho → -3.60977 - 0.188404 i}, {rho → -3.60977 + 0.188404 i},
 {rho → 0.00976686 - 0.352002 i}, {rho → 0.00976686 + 0.352002 i}}
```

→ *No solution!*

- **[3] For fixed ρ = 0.2 and fixed $N_{||}$ = 0.5**

```
soloxc3 = Solve[q == oxc //. {rho → 0.2, Npar → 0.5}, ne0]
```

```
{{ne0 → 1.02193}}
```

```
oxcne0 = ne0 /. soloxc3[[1]];
```

- **[4] For fixed ρ = 0.2 and fixed n_{e0}=1.0 x $10^{20}\, m^{-3}$**

```
soloxc4 = Solve[q == oxc //. {rho → 0.2, ne0 → 1}, Npar]
```

```
{{Npar → -1.88051}, {Npar → -0.531771}, {Npar → 0.531771}, {Npar → 1.88051}}
```

→ *No solution!*

- **Thus, for fixed $N_{||}$ of 0.5, the maximum density > 1.0 x $10^{20}\, m^{-3}$ is required to have O-X mode conversion at ρ = 0.2.**
  For the maximum density of 1.0 x $10^{20}\, m^{-3}$, the parallel refractive index, $N_{||}$ > 0.5 to have O-X mode conversion at ρ = 0.2.

- **<u>O-X Propagation Plot for N|| = 0.5</u>**

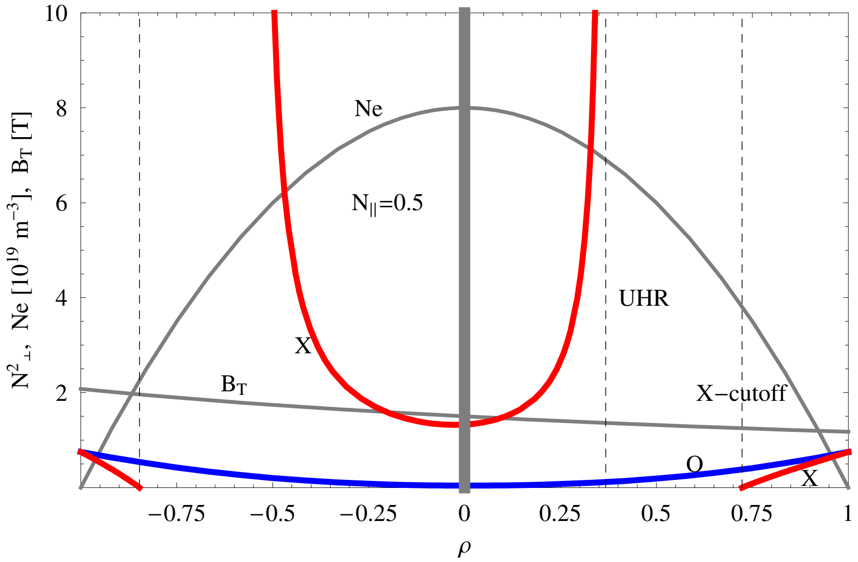
```
OX_temp1 = Plot[{ Nppsq //. {ne0 → 0.8, Npar → 0.5}, Npnsq //. {ne0 → 0.8, Npar → 0.5}},
  {rho, -1, 1}, PlotStyle → {{Thickness[0.008], RGBColor[0, 0, 1]},
    {Thickness[0.008], RGBColor[1, 0, 0]}}, PlotRange → {0, 5}]
```

```
Omode1 = Plot[Nppsq //. {ne0 → 0.8, Npar → 0.5},
  {rho, -1, 1}, PlotStyle → {Thickness[0.008], RGBColor[0, 0, 1]}]
```

```
Xmode1 = Plot[ Npnsq //. {ne0 → 0.8, Npar → 0.5},
  {rho, xcutrho1, 1}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]
```

```
Xmode2 = Plot[ Npnsq //. {ne0 → 0.8, Npar → 0.5},
  {rho, uhrrho2, uhrrho1}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]
```

```
Xmode3 = Plot[Npnsq //. {ne0 → 0.8, Npar → 0.5},
  {rho, -1, xcutrho2}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]

OXWavePlot = Show[{Omode1, Xmode1, Xmode2, Xmode3}, PlotRange → {0, 5}]

Show[{NePlot, BTPlot, OXWavePlot},
 Graphics[{{Dashing[{0.01, 0.01}], Line[{{xcutrho1, 0}, {xcutrho1, 10}}]},
   {Dashing[{0.01, 0.01}], Line[{{xcutrho2, 0}, {xcutrho2, 10}}]},
   {Dashing[{0.01, 0.01}], Line[{{uhrrho1, 0}, {uhrrho1, 10}}]},
   {Thickness[0.015], GrayLevel[0.5], Line[{{ecrrho, 0}, {ecrrho, 10}}]}}],
 Graphics[{Text["N||=0.5", {-0.2, 6}], Text["O", {0.6, 0.5}],
   Text["UHR", {uhrrho1 + 0.1, 4}], Text["Ne", {-0.25, 8.}], Text["BT", {-0.6, 2.2}],
   Text["X", {0.9, 0.2}], Text["X-cutoff", {xcutrho1, 2}], Text["X", {-0.42, 3}]}],
 Frame → True, FrameLabel → {"ρ", "N2⊥,  Ne [10^19 m^-3],  BT [T]"},
 PlotRange → {{-1, 1}, {0, 10}}]
```




```
- Graphics -
```

2.2 O-X propagation for 2nd harmonic resonance for KSTAR tokamak with high density

Propagation of 84-GHz Microwave in KSTAR tokamak for Second Harmonic Resonance with high plasma density

- **Electron density in unit of 10^20 m^-3**
  Electron temperuture in unit of keV
  Toroidal magnetic field in unit of Tesla
  All frequencies in unit of GHz

  KSTAR major radius: 1.8 m
  KSTAR plasma minor radius: 0.5 m
  KSTAR toroidal magnetic field, B0: 1.5 T
  KSTAR ECH system frequency: 84 GHz

```
Clear["Global`*"]

Off[General::spell];
Off[General::spell1];

a = 0.5;
R0 = 1.8;
f = 84.0;
bz0 = 1.5;
te0 = 10.;
ne1 = 0.;
Nh = 2;
Nnu = 1.;
Tnu = 1;
Az = 1;
massr = 2000. Az;
sc = 3 10^8;
mc2 = 511.0;
```

```
ne = ne0 ((1 - rho^2))^Nnu + ne1;
te = te0 ((1 - rho^2))^Tnu;
ve = sc Sqrt[2 te / mc2];
bz = bz0 / (1 + (a / R0) rho);
fce = 28.0 bz;
fci = fce / massr;
fpe = 90.0 Sqrt[ne];
fpi = fpe / Sqrt[massr];
w = 2.0 Pi f;
wce = 2.0 Pi fce;
wpe = 2.0 Pi fpe;
wci = 2.0 Pi fci;
wpi = 2.0 Pi fpi;
```

- **SS = 1 -** $\omega^2_{pe} / (\omega^2 - \omega^2_{ce})$
  DD = $(-\omega^2_{pe}/(\omega^2 - \omega^2_{ce}))\ (\omega_{ce}/\omega)$
  PP = 1 - ω^2_{pe}/ω^2

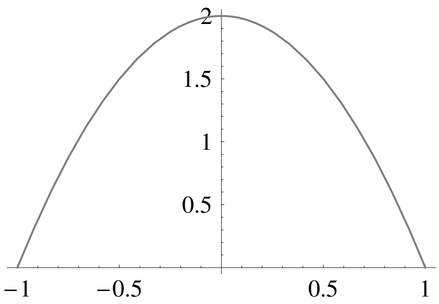
  If define
  q = $\omega^2_{pe}/\ \omega^2$
  u = $\omega^2_{ce}/\ \omega^2$

```
q = wpe^2 / w^2;
u = wce^2 / w^2;

SS = 1 - q / (1 - u);
DD = -q / (1 - u) Sqrt[u];
PP = 1 - q;

AA = SS;
BB = - (SS + PP) (SS - Npar^2) + DD^2;
CC = PP ((SS - Npar^2)^2 - DD^2);

Disc = BB BB - 4 AA CC;

Nppsq = (-BB + Sqrt[Disc]) / (2.0 AA);
Npnsq = (-BB - Sqrt[Disc]) / (2.0 AA);
```

▪ + sign : O-mode , - sign : X-mode

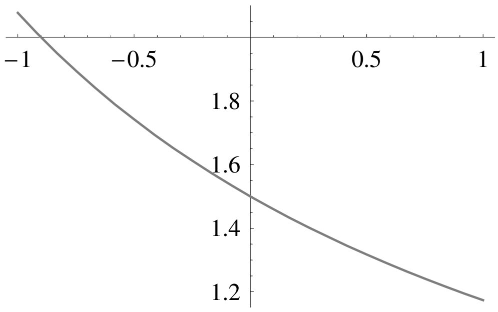
▪ Density plot (low density)

```
NePlot =
 Plot[2 ne /. ne0 → 1, {rho, -1, 1}, PlotStyle → {Thickness[0.005], GrayLevel[0.5]}]
```




```
- Graphics -
```

```
BTPlot = Plot[ bz, {rho, -1, 1}, PlotStyle → {Thickness[0.005], GrayLevel[0.5]}]
```




```
- Graphics -
```

▪ Cutoff and Resonances

O-mode cutoff: q = 1 (⟸ P = 0)

X-mode cutoff: $q = (1-\sqrt{u})(1-N_{\parallel}^2)$ ($\Longleftarrow (S-N_{\parallel}^2)^2 - D^2 = 0$)

Upper Hybrid Resonance: q = 1 - u ($\Longleftarrow \omega^2 = \omega^2_{pe} + \omega^2_{ce}$)

Electron Cyclotron Resonance: $\sqrt{u} = 1$ ($\Longleftarrow \omega = N\,\omega_{ce}$)

O-X conversion: $q = 1 + u\,[(1-N_{\parallel}^2)/(2\,N_{\parallel})]^2$ ($\Longleftarrow B^2 - 4\,AC = 0$)

```
Clear[Npar];
```

```
ocut = 1;
xcut = (1 - Sqrt[u]) (1 - Npar^2);
uhr = (1 - u);
ecr = Nh Sqrt[ u] - 1;
oxc = 1 + u ((1 - Npar^2) / (2 Npar)) ^2;
```

■ O-Mode Cutoff position (for maximum density of 0.8 x 10^20 m^-3)

```
solocut = Solve[q == ocut //. ne0 → 1, rho]
```

```
{{rho → -0.359011}, {rho → 0.359011}}
```

```
ocutrho1 = rho /. solocut[[2]];
ocutrho2 = rho /. solocut[[1]];
```

■ X-mode Cutoff position for $N_{\|} = 0.5$

```
solxcut = Solve[q == xcut //. {ne0 → 1, Npar → 0.5}, rho]
```

```
{{rho → -3.68869}, {rho → -3.50128}, {rho → -0.88288}, {rho → 0.784159}}
```

```
xcutrho1 = rho /. solxcut[[4]];
xcutrho2 = rho /. solxcut[[3]];
```

■ Upper Hybrid Resonance position

```
soluhr = Solve[q == uhr /. ne0 → 1, rho]
```

```
{{rho → -4.01963}, {rho → -3.04426}, {rho → -0.677797}, {rho → 0.541688}}
```

```
uhrrho1 = rho /. soluhr[[4]];
uhrrho2 = rho /. soluhr[[3]];
```

■ ECR position

```
solecr = Solve[ecr == 0, rho]
```

```
{{rho → -7.2}, {rho → 0.}}
```

```
ecrrho = rho /. solecr[[2]];
```

■ O-X Conversion

■ [1] For fixed centeral density, n_{e0} =1x $10^{20}\,m^{-3}$ and fixed $N_{\|} = 0.5$

```
soloxc1 = Solve[q == oxc //. {ne0 → 1, Npar → 0.5}, rho]
```

```
{{rho → -3.63314 - 0.345317 i},
 {rho → -3.63314 + 0.345317 i}, {rho → -0.0523865}, {rho → 0.11867}}
```

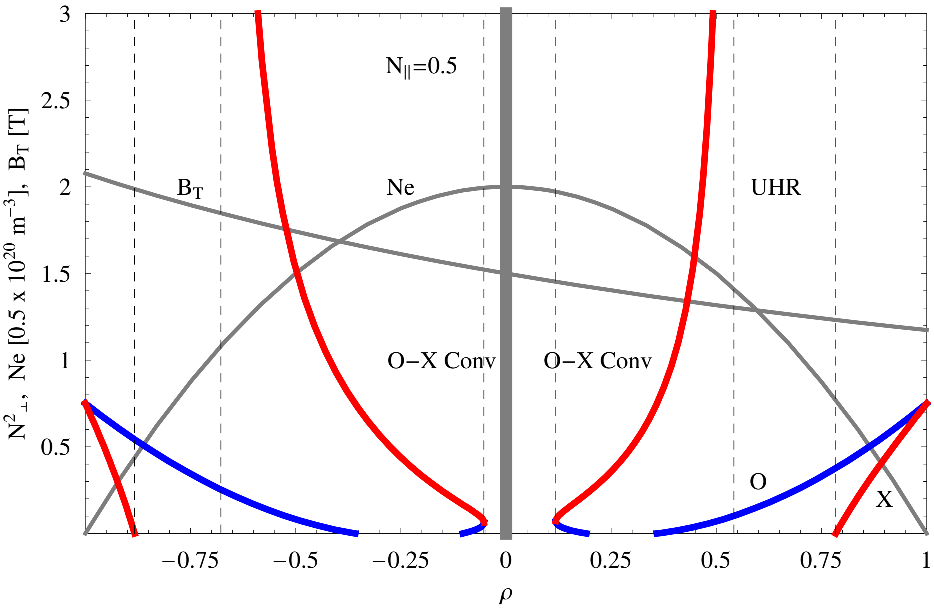
```
oxcrho1 = rho /. soloxc1[[4]];
oxcrho2 = rho /. soloxc1[[3]];
```

■ For the maximum density of 1.0 x $10^{20}\ m^{-3}$, the parallel refractive index, $N_{\parallel}$ > 0.5 to have O-X mode conversion at ρ ~ 0.1.

■ O-X Propagating for N|| = 0.5 (O-X conversion occurrence)

```
OXconv_temp2 = Plot[{Nppsq //. {ne0 → 1, Npar → 0.5}, Npnsq //. {ne0 → 1, Npar → 0.5}},
  {rho, -1, 1}, PlotStyle → {{Thickness[0.008], RGBColor[0, 0, 1]},
    {Thickness[0.008], RGBColor[1, 0, 0]}}, PlotRange → {0, 5}]

Omode21 = Plot[Nppsq //. {ne0 → 1, Npar → 0.5},
  {rho, oxcrho1, 1}, PlotStyle → {Thickness[0.008], RGBColor[0, 0, 1]}]

Omode22 = Plot[Nppsq //. {ne0 → 1, Npar → 0.5},
  {rho, oxcrho2, -1}, PlotStyle → {Thickness[0.008], RGBColor[0, 0, 1]}]

Xmode21 = Plot[ Npnsq //. {ne0 → 1, Npar → 0.5},
  {rho, xcutrho1, 1}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]

Xmode22 = Plot[ Npnsq //. {ne0 → 1, Npar → 0.5},
  {rho, oxcrho1, uhrrho1 - 0.013}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]

Xmode23 = Plot[ Npnsq //. {ne0 → 1, Npar → 0.5},
  {rho, oxcrho2, uhrrho2 + 0.02}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]

Xmode24 = Plot[ Npnsq //. {ne0 → 1, Npar → 0.5},
  {rho, -1, xcutrho2}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]

OXWavePlot2 = Show[{Omode21, Omode22, Xmode21, Xmode22, Xmode23, Xmode24}]
```

```
Show[{NePlot, BTPlot, OXWavePlot2},
 Graphics[{{Dashing[{0.01, 0.01}], Line[{{xcutrho2, 0}, {xcutrho2, 3}}]},
   {Dashing[{0.01, 0.01}], Line[{{xcutrho1, 0}, {xcutrho1, 3}}]},
   {Dashing[{0.01, 0.01}], Line[{{oxcrho2, 0}, {oxcrho2, 3}}]},
   {Dashing[{0.01, 0.01}], Line[{{oxcrho1, 0}, {oxcrho1, 3}}]},
   {Dashing[{0.01, 0.01}], Line[{{uhrrho1, 0}, {uhrrho1, 3}}]},
   {Dashing[{0.01, 0.01}], Line[{{uhrrho2, 0}, {uhrrho2, 3}}]},
   {Thickness[0.015], GrayLevel[0.5], Line[{{ecrrho, 0}, {ecrrho, 3}}]}}],
 Graphics[{Text["N||=0.5", {-0.2, 2.7}], Text["O-X Conv", {oxcrho1 + 0.1, 1}],
   Text["Ne", {-0.25, 2.}], Text["BT", {-0.75, 2}],
   Text["O-X Conv", {oxcrho2 - 0.1, 1}], Text["O", {0.6, 0.3}],
   Text["UHR", {uhrrho1 + 0.1, 2}], Text["X", {0.9, 0.2}]}],
 Frame → True, FrameLabel → {"ρ", "N2⊥,  Ne [0.5 x 10^20 m^-3],  BT [T]"},
 PlotRange → {{-1, 1}, {0, 3}}]
```



- Graphics -

2.3 O-X-B heating

Propagation of 84-GHz Microwave in KSTAR tokamak for Fundamental Harmonic Resonance with high density plasma

- **Electron density in unit of 10^20 m^-3**
  Electron temperuture in unit of keV
  Toroidal magnetic field in unit of Tesla
  All frequencies in unit of GHz

  KSTAR major radius: 1.8 m
  KSTAR plasma minor radius: 0.5 m
  KSTAR toroidal magnetic field, B0: 3.5 T
  KSTAR ECH system frequency: 84 GHz

```
Clear["Global`*"]
```

```
Off[General::spell];
Off[General::spell1];
```

```
a = 0.5;
R0 = 1.8;
f = 84.0;
bz0 = 3.5;
te0 = 10.;
ne1 = 0.;
Nh = 1;
Nnu = 1.;
Tnu = 1;
Az = 1;
massr = 2000. Az;
sc = 3 10^8;
mc2 = 511.0;
```

```
ne = ne0 ((1 - rho^2))^Nnu + ne1;
te = te0 ((1 - rho^2))^Tnu;
ve = sc Sqrt[2 te / mc2];
bz = bz0 / (1 + (a / R0) rho);
fce = 28.0 bz;
fci = fce / massr;
fpe = 90.0 Sqrt[ne];
fpi = fpe / Sqrt[massr];
w = 2.0 Pi f;
wce = 2.0 Pi fce;
wpe = 2.0 Pi fpe;
wci = 2.0 Pi fci;
wpi = 2.0 Pi fpi;
```

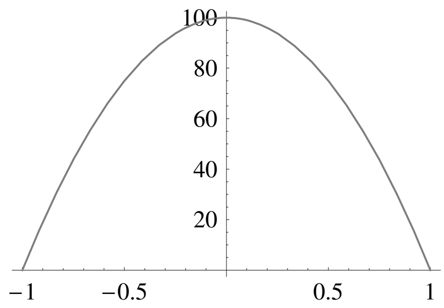
- **SS = 1 -** $\omega^2_{pe} / (\omega^2 - \omega^2_{ce})$
  DD = $(-\omega^2_{pe}/(\omega^2 - \omega^2_{ce}))\ (\omega_{ce}/\omega)$
  PP = 1 - ω^2_{pe}/ω^2

  If define
  q = $\omega^2_{pe}/\ \omega^2$
  u = $\omega^2_{ce}/\ \omega^2$

```
q = wpe^2 / w^2;
u = wce^2 / w^2;

SS = 1 - q / (1 - u);
DD = -q / (1 - u) Sqrt[u];
PP = 1 - q;

AA = SS;
BB = - (SS + PP) (SS - Npar^2) + DD^2;
CC = PP ((SS - Npar^2)^2 - DD^2);

Disc = BB BB - 4 AA CC;

Nppsq = (-BB + Sqrt[Disc]) / (2.0 AA);
Npnsq = (-BB - Sqrt[Disc]) / (2.0 AA);
```

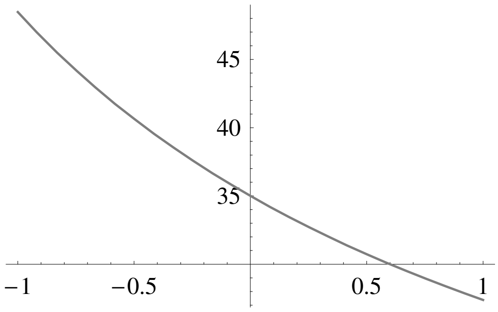
▪ + sign : O-mode , - sign : X-mode

```
NePlot =
 Plot[100 ne /. ne0 → 1, {rho, -1, 1}, PlotStyle → {Thickness[0.005], GrayLevel[0.5]}]
```



```
- Graphics -
```

```
BTPlot = Plot[10 bz, {rho, -1, 1}, PlotStyle → {Thickness[0.005], GrayLevel[0.5]}]
```



```
- Graphics -
```

■ Cutoff and Resonances

O-mode cutoff: q = 1 **(⟸ P = 0)**

X-mode cutoff: $q = (1-\sqrt{u})(1-N_{\parallel}^2)$ **(⟸ $(S-N_{\parallel}^2)^2 - D^2 = 0$)**

Upper Hybrid Resonance: q = 1 - u **(⟸ $\omega^2 = \omega^2_{pe} + \omega^2_{ce}$)**

Electron Cyclotron Resonance: $\sqrt{u} = 1$ **(⟸ $\omega = \omega_{ce}$)**

O-X conversion: $q = 1 + u\,[(1-N_{\parallel}^2)/(2\,N_{\parallel})]^2$ **(⟸ $B^2 - 4\,AC = 0$)**

```
Clear[Npar];
```

```
ocut = 1;
xcut = (1 - Sqrt[u]) (1 - Npar^2);
uhr = (1 - u);
ecr = Sqrt[u] - 1;
oxc = 1 + u ((1 - Npar^2) / (2 Npar)) ^2;
```

■ O-Mode Cutoff position

```
solocut = Solve[q == ocut //. ne0 → 1, rho]
```

```
{{rho → -0.359011}, {rho → 0.359011}}
```

```
ocutrho1 = rho /. solocut[[1]];
ocutrho2 = rho /. solocut[[2]];
```

■ X-mode Cutoff position for $N_{\shortparallel} = 0.5$

```
solxcut = Solve[q == xcut //. {ne0 → 1, Npar → 0.5}, rho]
```

```
{{rho → -3.79521}, {rho → -3.34727}, {rho → -1.22572}, {rho → 0.972991}}
```

```
xcutrho = rho /. solxcut[[4]];
```

■ Upper Hybrid Resonance position

```
soluhr = Solve[q == uhr /. ne0 → 1, rho]
```

```
{{rho → -4.47818}, {rho → -1.82883 - 0.848917 i},
 {rho → -1.82883 + 0.848917 i}, {rho → 0.935832}}
```

```
uhrrho = rho /. soluhr[[4]];
```

■ ECR position

```
solecr = Solve[ecr == 0 /. ne0 → 1, rho]
```

```
{{rho → -7.8}, {rho → 0.6}}
```

```
ecrrho = rho /. solecr[[2]];
```

■ O-X Conversion

■ [1] For fixed centeral density, n_{e0}=1x 10^{20} m^{-3} and fixed $N_{\shortparallel} = 0.5$

```
soloxc1 = Solve[q == oxc //. {ne0 → 1, Npar → 0.5}, rho]
```

```
{{rho → -3.75345 - 0.755581 i}, {rho → -3.75345 + 0.755581 i},
 {rho → 0.153449 - 0.672412 i}, {rho → 0.153449 + 0.672412 i}}
```

$\rightarrow$ *No solution!*

- **[2] For fixed centeral density, n_{e0}=1 x $10^{20}\,m^{-3}$ and fixed $N_{\|}$ = 0.8**

```
soloxc2 = Solve[q == oxc //. {ne0 → 1, Npar → 0.8}, rho]
```

```
{{rho → -3.61662 - 0.243947 i},
 {rho → -3.61662 + 0.243947 i}, {rho → -0.244533}, {rho → 0.277768}}
```

```
oxcrho1 = rho /. soloxc2[[3]];
oxcrho2 = rho /. soloxc2[[4]];
```

- **[3] For fixed ρ = 0.2 and fixed $N_{\|}$ = 0.5**

```
soloxc3 = Solve[q == oxc //. {rho → 0.2, Npar → 0.5}, ne0]
```

```
{{ne0 → 1.53094}}
```

```
oxcne0 = ne0 /. soloxc3[[1]];
```

- **[4] For fixed ρ = 0.2 and fixed n_{e0}=1 x $10^{20}\,m^{-3}$**

```
soloxc4 = Solve[q == oxc //. {rho → 0.2, ne0 → 1}, Npar]
```

```
{{Npar → -1.32994}, {Npar → -0.751912}, {Npar → 0.751912}, {Npar → 1.32994}}
```

```
oxcnpar = Npar /. soloxc4[[3]];
```

- **Thus, for fixed $N_{\|}$ of 0.5, the maximum density > 1.53 x $10^{20}\,m^{-3}$ is required to have O-X mode conversion at ρ = 0.2.
For the maximum density of 1.0 x $10^{20}\,m^{-3}$, the parallel refractive index, $N_{\|}$ > 0.75 to have O-X mode conversion at ρ = 0.2. However, for large $N_{\|}$, there**

- **X-mode Cutoff position for $N_{\|}$ = 0.8**

```
solxcut = Solve[q == xcut //. {ne0 → 1, Npar → 0.8}, rho]
```

```
{{rho → -3.70122}, {rho → -3.48506}, {rho → -1.10163}, {rho → 0.986692}}
```

```
xcutrho2 = rho /. solxcut[[4]];
```

```
<< Graphics`Graphics`
```

```
$TextStyle = {FontFamily → "Times", FontSize → 14};
```

▪ Electron Bernstein Mode

$$\frac{\alpha(Q,b_e)}{b_e} = \frac{1}{\omega_{pe}^2/\Omega_e^2}$$

where

$$\frac{\alpha(Q,b_e)}{b_e} = \frac{1}{Q^2-1^2} + \frac{1\cdot 3\, b_e}{(Q^2-1^2)(Q^2-2^2)} + \frac{1\cdot 3\cdot 5\, b_e^2}{(Q^2-1^2)(Q^2-2^2)(Q^2-3^2)} + \cdots$$

with

$$Q = \frac{\omega}{\Omega_e},\; b_e = \frac{k_\perp^2 V_e^2}{\square}$$

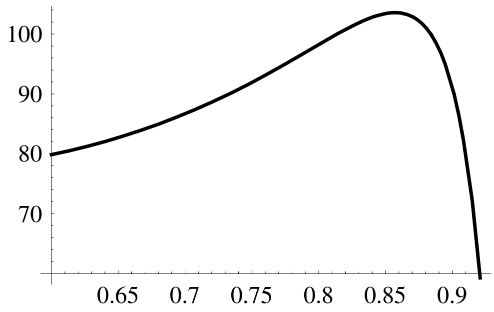
```
kperp2 = (10. w / 3.) ^2 Npb2;
be = kperp2 ve^2 / (2. wce^2 10^18);
Qe = w / wce;
XX = wpe^2 / wce^2;

alphaovbe = 1 / (Qe^2 - 1) + 3 be / ((Qe^2 - 1) (Qe^2 - 2^2));

solNpb = Solve[alphaovbe == 1 / XX, Npb2]
```

$$\left\{\left\{Npb2 \to \left(23.1843\,(-4. + 0.734694\,(1. + 0.277778\,rho)^2)\,(-1. + 0.734694\,(1. + 0.277778\,rho)^2)\left(-\frac{1.}{-1. + 0.734694\,(1. + 0.277778\,rho)^2} + \frac{1.18568}{(1. + 0.277778\,rho)^2\left(0. + ne0\,(1. - 1.\,rho^2)^{1.}\right)}\right)\right)\Big/\left((1. + 0.277778\,rho)^2\,(1. - 1.\,rho^2)\right)\right\}\right\}$$

```
NEBW = Npb2 /. solNpb[[1]]
```

$$\left(23.1843\,(-4. + 0.734694\,(1. + 0.277778\,rho)^2)\,(-1. + 0.734694\,(1. + 0.277778\,rho)^2)\left(-\frac{1.}{-1. + 0.734694\,(1. + 0.277778\,rho)^2} + \frac{1.18568}{(1. + 0.277778\,rho)^2\left(0. + ne0\,(1. - 1.\,rho^2)^{1.}\right)}\right)\right)\Big/\left((1. + 0.277778\,rho)^2\,(1. - 1.\,rho^2)\right)$$

```
EBWPlot = Plot[NEBW /. ne0 → 1, {rho, ecrrho, uhrrho - 0.015},
  PlotStyle → {Thickness[0.008]}, PlotRange → Automatic]
```

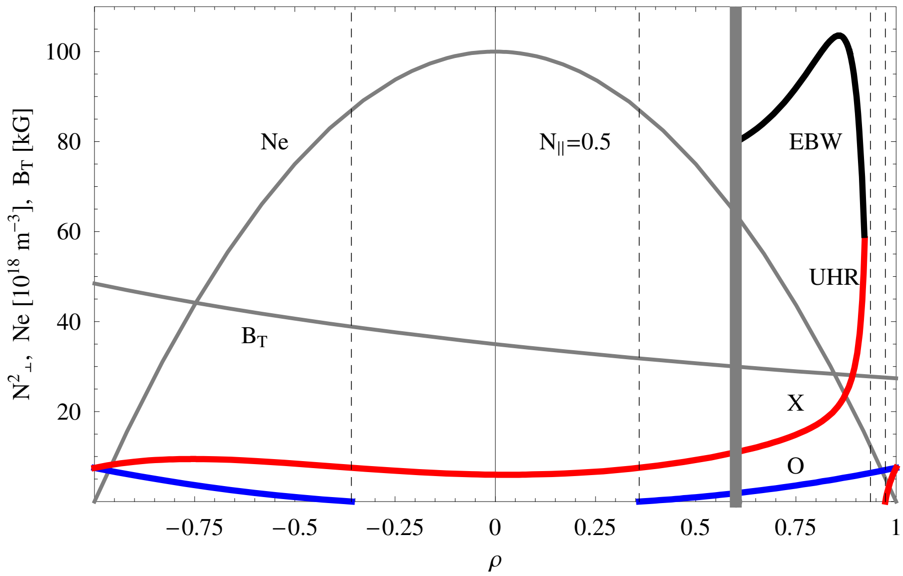



```
- Graphics -
```

■ X-EBW Heating Scheme

```
Omode1 = Plot[10 Nppsq //. {ne0 → 1, Npar → 0.5},
  {rho, ecrrho, 1}, PlotStyle → {Thickness[0.008], RGBColor[0, 0, 1]}]

Omode2 = Plot[10 Npnsq //. {ne0 → 1, Npar → 0.5},
  {rho, ecrrho, ocutrho2}, PlotStyle → {Thickness[0.008], RGBColor[0, 0, 1]}]

Omode3 = Plot[10 Npnsq //. {ne0 → 1, Npar → 0.5},
  {rho, -1, ocutrho2}, PlotStyle → {Thickness[0.008], RGBColor[0, 0, 1]}]

Xmode1 = Plot[10 Npnsq //. {ne0 → 1, Npar → 0.5},
  {rho, xcutrho, 1}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]

Xmode2 = Plot[10 Npnsq //. {ne0 → 1, Npar → 0.5},
  {rho, ecrrho, uhrrho - 0.014}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]

Xmode3 = Plot[10 Nppsq //. {ne0 → 1, Npar → 0.5},
  {rho, -1, ecrrho}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]

OXWavePlot = Show[{Omode1, Omode2, Omode3, Xmode1, Xmode2, Xmode3}]
```

```
Show[{NePlot, BTPlot, OXWavePlot, EBWPlot},
 Graphics[{{Dashing[{0.01, 0.01}], Line[{{ocutrho1, 0}, {ocutrho1, 110}}]},
   {Dashing[{0.01, 0.01}], Line[{{ocutrho2, 0}, {ocutrho2, 110}}]},
   {Dashing[{0.01, 0.01}], Line[{{xcutrho, 0}, {xcutrho, 110}}]},
   {Dashing[{0.01, 0.01}], Line[{{uhrrho, 0}, {uhrrho, 110}}]},
   {Thickness[0.015], GrayLevel[0.5], Line[{{ecrrho, 0}, {ecrrho, 118}}]}}],
 Graphics[{Text["N||=0.5", {0.2, 80}], Text["O", {0.75, 8}],
   Text["UHR", {uhrrho - 0.09, 50}], Text["Ne", {-0.55, 80}],
   Text["BT", {-0.6, 37}], Text["X", {0.75, 22}], Text["EBW", {0.8, 80}]}],
 Frame → True, FrameLabel → {"ρ", "N2⊥,  Ne [10^18 m^-3],  BT [kG]"},
 PlotRange → {{-1, 1}, {0, 110}}]
```

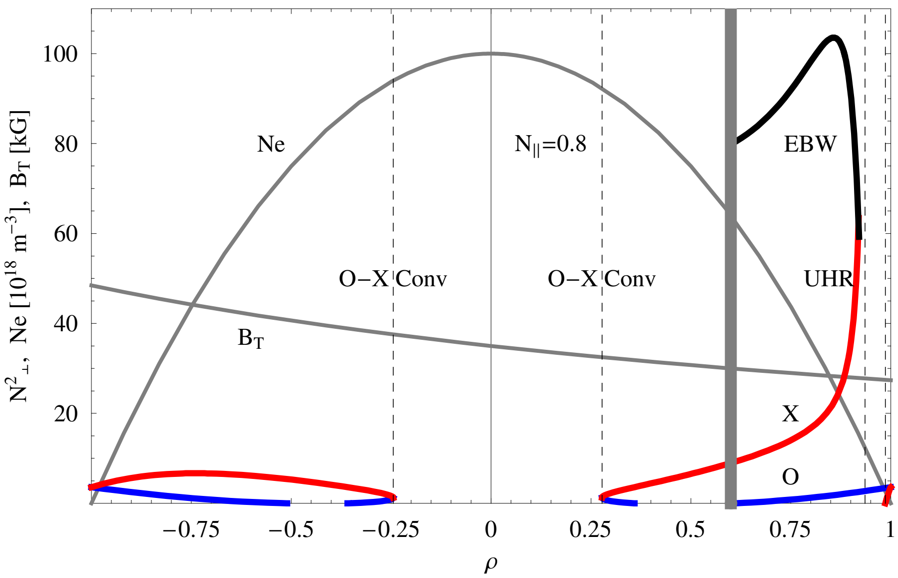



```
- Graphics -
```

■ O-X-EBW Heating Scheme

```
OXconv_temp2 =
 Plot[{10 Nppsq //. {ne0 → 1, Npar → 0.8}, 10 Npnsq //. {ne0 → 1, Npar → 0.8}},
  {rho, -1, 1}, PlotStyle → {{Thickness[0.008], RGBColor[0, 0, 1]},
    {Thickness[0.008], RGBColor[1, 0, 0]}}, PlotRange → {0, 100}]

Omode21 = Plot[10 Nppsq //. {ne0 → 1, Npar → 0.8},
  {rho, ecrrho, 1}, PlotStyle → {Thickness[0.008], RGBColor[0, 0, 1]}]

Omode22 = Plot[10 Npnsq //. {ne0 → 1, Npar → 0.8},
  {rho, ecrrho, oxcrho2}, PlotStyle → {Thickness[0.008], RGBColor[0, 0, 1]}]

Omode23 = Plot[10 Npnsq //. {ne0 → 1, Npar → 0.8},
  {rho, -1, oxcrho1}, PlotStyle → {Thickness[0.008], RGBColor[0, 0, 1]}]

Xmode21 = Plot[10 Npnsq //. {ne0 → 1, Npar → 0.8},
  {rho, xcutrho2, 1}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]
```

```
Xmode22 = Plot[10 Npnsq //. {ne0 → 1, Npar → 0.8},
  {rho, ecrrho, uhrrho - 0.016}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]

Xmode23 = Plot[10 Nppsq //. {ne0 → 1, Npar → 0.8},
  {rho, oxcrho2, ecrrho}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]

Xmode24 = Plot[10 Nppsq //. {ne0 → 1, Npar → 0.8},
  {rho, -1, oxcrho1}, PlotStyle → {Thickness[0.008], RGBColor[1, 0, 0]}]

OXWavePlot2 = Show[{Omode21, Omode22, Omode23, Xmode21, Xmode22, Xmode23, Xmode24}]

Show[{NePlot, BTPlot, OXWavePlot2, EBWPlot},
 Graphics[{{Dashing[{0.01, 0.01}], Line[{{xcutrho2, 0}, {xcutrho2, 110}}]},
   {Dashing[{0.01, 0.01}], Line[{{oxcrho2, 0}, {oxcrho2, 110}}]},
   {Dashing[{0.01, 0.01}], Line[{{oxcrho1, 0}, {oxcrho1, 110}}]},
   {Dashing[{0.01, 0.01}], Line[{{uhrrho, 0}, {uhrrho, 110}}]},
   {Thickness[0.015], GrayLevel[0.5], Line[{{ecrrho, 0}, {ecrrho, 118}}]}}],
 Graphics[{Text["N||=0.8", {0.15, 80}], Text["O-X Conv", {oxcrho1, 50}],
   Text["Ne", {-0.55, 80}], Text["BT", {-0.6, 37}],
   Text["O-X Conv", {oxcrho2, 50}], Text["O", {0.75, 6}],
   Text["UHR", {uhrrho - 0.09, 50}], Text["X", {0.75, 20}], Text["EBW", {0.8, 80}]}],
 Frame → True, FrameLabel → {"ρ", "N^2_⊥,  Ne  [10^18 m^-3],  BT  [kG]"},
 PlotRange → {{-1, 1}, {0, 110}}]
```



- Graphics -