A Calculation of S_x

$$S_x = \frac{1}{4\mu_0}\mathrm{Re}\left[\vec{E}\times\vec{B}^* + \frac{1}{2}\vec{E}^*\cdot\frac{\partial}{\partial\vec{N}}\overleftrightarrow{\epsilon}\cdot\vec{E}\right]_x$$

$$\begin{aligned}\text{since } \vec{\nabla}\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t}\\ \rightarrow\ ik_xE_y &= (-i\omega)B_z\\ \therefore\ B_z &= \frac{k_x}{\omega}E_y = \frac{N_x}{c}E_y\end{aligned}$$

and,

$$ik_zE_x - ik_xE_z = i\omega B_y,\ \ \frac{1}{c}(N_zE_x - N_xE_z) = B_y$$

Let $\frac{\partial}{\partial\vec{N}}\overleftrightarrow{\epsilon}\cdot\vec{E} = X$

$$S_x = \frac{1}{4\mu_0}\mathrm{Re}\left[(E_yB_z^* - E_zB_y^*) + \frac{1}{2}(E_x^*X_{xx} + E_y^*X_{yx} + E_z^*X_{zx})\right]$$

But,

A:

$$\begin{aligned}X &= \frac{\partial}{\partial\vec{N}}\overleftrightarrow{\epsilon}\cdot\vec{E}\\ &= \hat{x}\frac{\partial}{\partial N_x}\left[\hat{x}(\epsilon_{xx}E_x + \epsilon_{xy}E_y) + \hat{y}(\epsilon_{yx}E_x + \epsilon_{yy}E_y + \hat{z}\epsilon_{zz}E_z\right]\\ &+ \hat{z}\frac{\partial}{\partial N_z}\left[\hat{x}(\epsilon_{xx}E_x + \epsilon_{xy}E_y) + \hat{y}(\epsilon_{yx}E_x + \epsilon_{yy}E_y + \hat{z}\epsilon_{zz}E_z\right]\end{aligned}$$

$$\begin{cases}X_{xx} &= \frac{\partial}{\partial N_x}(\epsilon_{xx}E_x + \epsilon_{xy}E_y)\\ X_{yx} &= 0\\ X_{zx} &= \frac{\partial}{\partial N_z}(\epsilon_{xx}E_x + \epsilon_{xy}E_y)\end{cases}$$

before,

$$E_x = \frac{S-N^2}{D}iE_y,\ E_z = -\frac{N_xN_z}{P-N_x^2}\frac{S-N^2}{D}iE_y$$

Then,

$$X_{xx} = \frac{\partial}{\partial N_x}\left(S\frac{S-N^2}{D} - D\right)iE_y = -\frac{S}{D}(2N_x)iE_y$$
$$X_{zx} = \frac{\partial}{\partial N_z}\left(S\frac{S-N^2}{D} - D\right)iE_y = -\frac{S}{D}(2N_z)iE_y$$

Therefore,

$$\begin{aligned}E_x^*X_{xx} &= \frac{S-N^2}{D}(-iE_y^*)\left(-\frac{S}{D}\right)2N_xiE_y = -\frac{2S}{D^2}(S-N^2)N_x|E_y|^2\\ E_y^*X_{yx} &= 0\\ E_z^*X_{zx} &= -\frac{N_xN_z}{P-N_x^2}\frac{S-N^2}{D}(-iE_y^*)\left(-\frac{S}{D}\right)2N_ziE_y = \frac{2N_xN_z^2}{P-N_x^2}\frac{S(S-N^2)}{D^2}|E_y|^2\end{aligned}$$

$$
\begin{aligned}
\implies \quad & \frac{1}{2}(E_x^* X_{xx} + E_y^* X_{yx} + E_z^* X_{zx}) \\
= \quad & \left[-\frac{S(S-N^2)}{D^2} + \frac{N_z^2}{P-N_x^2} \frac{S}{D^2}(S-N^2) \right] N_x |E_y|^2 \\
= \quad & N_x |E_y|^2 \left[-\frac{S(S-N^2)}{D^2} \right] \left(1 - \frac{N_z^2}{P-N_x^2} \right)
\end{aligned}
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B:

$$
\begin{aligned}
E_y B_z^* - E_z B_y^* &= \frac{1}{c} \left[E_y N_x E_y^* - E_z (N_z E_x^* - N_x E_z^*) \right] \\
&= \frac{1}{c} \left[N_x |E_y|^2 + \frac{N_x N_z^2}{P-N_x^2} \left(\frac{S-N^2}{D} \right)^2 |E_y|^2 + N_x \frac{N_x^2 N_z^2}{(P-N_x^2)^2} \frac{(S-N^2)^2}{D^2} |E_y|^2 \right] \\
&= \frac{1}{c} N_x |E_y|^2 \left(1 + \frac{(S-N^2)^2 N_z^2}{D^2 (P-N_x^2)} + \frac{(S-N^2)^2 N_x^2 N_z^2}{D^2 (P-N_x^2)^2} \right) \\
&= \frac{1}{c} N_x |E_y|^2 \frac{D^2 (P-N_x^2)^2 + (S-N^2)^2 (P-N_x^2) N_z^2 + (S-N^2)^2 N_x^2 N_z^2}{D^2 (P-N_x^2)^2} \\
&= \frac{1}{c} N_x |E_y|^2 \frac{D^2 (P-N_x^2)^2 + (S-N^2)^2 (P N_z^2 - N_x^2 N_z^2 + N_x^2 N_z^2)}{D^2 (P-N_x^2)^2} \\
&= \frac{1}{c} N_x |E_y|^2 \frac{D^2 (P-N_x^2)^2 + (S-N^2)^2 (P N_z^2)}{D^2 (P-N_x^2)^2}
\end{aligned}
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\begin{aligned}
\therefore \; S_x &= \frac{1}{4\mu_0 c} N_x |E_y|^2 \left[\frac{D^2 (P-N_x^2)^2 + (S-N^2)^2 P N_z^2}{D^2 (P-N_x^2)^2} - \frac{S}{D^2}(S-N^2) \frac{P-N^2}{P-N_x^2} \right] \\
&\simeq \frac{1}{4\mu_0 c} N_x |E_y|^2 \frac{D^2 (P-N_x^2)^2 + (S-N^2)^2 P N_z^2}{D^2 (P-N_x^2)^2}
\end{aligned}
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